Memorandum

TO: Lou Thompson and John Chalker—California High Speed Rail Peer Review Group

FROM: Jeff Buxbaum, David Kurth—Cambridge Systematics

Thierry Prate – Parsons Brinckerhoff

DATE: April 20, 2012

RE: California High Speed Rail Project
Ridership and Revenue Model Sensitivity Tests and Extreme Downside Scenario

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This memo covers two related topics:

1. A demonstration of the sensitivity of the travel demand model to specific conditions identified by members of the California High-Speed Rail Peer Review Group (PRG.)

2. Preparing a ridership and revenue forecast for an extreme downside scenario.

Our analysis of these is provided below. Model run summaries for the model runs that are referenced are in Appendix A.

Sensitivity Tests

CS prepared two sensitivity tests:

1. Assuming an average fleet fuel economy of 50 miles per gallon; and

2. Assuming reduced frequency on the Peninsula

Both sensitivity tests were based on model runs prepared for the draft 2012 Business Plan as documented in the CS Technical Memorandum dated October 19, 2011, specifically, model run 35, which represented the high case for the business plan.[1] We used the draft business plan model run as a base because it had been published at the time this work was initiated. The focus of the analysis is on the comparison between the two runs, rather than the ridership value of the sensitivity test itself, since the Revised Business Plan uses different assumptions. Since

[1] The Phase 1 High ridership and revenue forecasts used for the Draft Business Plan were based on Run 14b, however after the Plan was published, CS made a small technical correction. The corrected run was Run 35. We use the corrected run for the sensitivity test comparisons in this memo.

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these sensitivity tests compare outcomes from the Draft Business Plan, we use 2010 dollars for the comparison, since this was used in the Draft Business Plan.

Average Fleet Fuel Economy of 50 MPG (Run 49)

The Ridership and Revenue (R&R) model uses an operating cost per mile composed of fuel and non-fuel components. The cost per mile related to fuel is a function of both fuel economy and price. For the Draft Business Plan, we used a cost per mile of 25 cents in 2005$, composed of 16 cents for fuel (based on about $4.00 per gallon) and 9 cents for non-fuel. In this sensitivity test (and in the model runs for the Revised Business Plan) we kept the 9 cents per mile constant regardless of fuel price and efficiency and doubled the fuel component of cost to 50 miles per gallon. Table 1 provides the specifics

Table 1: Comparison of Auto Operating Cost Assumptions:
Run 35 and Run 48

	Draft Business Plan (Run 35)	50 MPG test (Run 49)
Price per gallon (2005$)	$3.80	$3.80
Miles per gallon	23.8	50.0
Fuel cost per mile (2005$)	$0.16	$0.08
Non-auto operating cost (2005$)	$0.09	$0.09
Total cost / mile (2005$)	$0.25	$0.17
Total cost / mile (2011$)	$0.29	$0.19

The overall effect of the higher fuel economy is a reduction in ridership of 16 percent and revenue of 19 percent from the original run (see Table 2). The larger impacts are on the longer distance movements (in the 18 to 24 percent range on revenue,) with lower impacts for intraregional movements.

Table 2: Year 2030 Annual Region-to-Region Ridership and Revenue (millions)
50 MPG Fuel Efficiency Sensitivity Test
Revenue in 2010 Dollars

	Market	Run 10-035 Phase 1		Run 12-049 Phase 1		Percentage Difference	
		HST Ridership	HST Revenues	HST Ridership	HST Revenues	HST Ridership	HST Revenues
1	LA basin - Sacramento	1.8	$143	1.3	$108	-24%	-24%
2	LA basin - San Diego	0.2	$6	0.1	$5	-18%	-15%
3	LA basin- Bay Area	8.5	$683	6.9	$559	-18%	-18%
4	Sacramento - Bay Area	0.0	$0	0.0	$0	0%	-15%
5	San Diego- Sacramento	0.0	$2	0.0	$2	0%	-2%
6	San Diego- Bay Area	1.9	$153	1.7	$134	-13%	-13%
7	Bay Area - San Joaquin Valley	5.5	$396	4.2	$307	-22%	-22%
8	San Joaquin Valley - LA basin	5.2	$362	4.1	$286	-21%	-21%
9	Sacramento - San Joaquin Valley	0.4	$32	0.3	$26	-17%	-19%
10	San Diego - San Joaquin Valley	0.1	$4	0.1	$4	-17%	-11%
11	within Bay Area Peninsula*	3.3	$59	3.0	$55	-7%	-8%
12	within North LA basin*	3.1	$85	2.9	$79	-7%	-7%
14	within South LA basin*	1.2	$28	1.1	$26	-6%	-6%
15	North LA - South LA*	2.8	$78	2.6	$72	-7%	-8%
18	within San Diego region	0.0	$0	0.0	$0	0%	0%
19	within San Joaquin Valley	0.5	$32	0.4	$26	-19%	-18%
20	Other	4.6	$293	3.7	$230	-20%	-21%
	Total	39.0	$2,357	32.6	$1,920	-16%	-19%
	within San Diego region	0.0	$0	0.0	$0	0%	0%
	within entire LA basin	7.1	$191	6.6	$177	-7%	-7%
	within entire MTC	3.3	$59	3.0	$55	-7%	-8%
	within other regions	0.0	$0	0.0	$0	0%	0%
	Total between regions	28.6	$2,107	22.9	$1,688	-20%	-20%

Reduced Frequency on Peninsula (Run 48)

This sensitivity test evaluates the implications of lower frequency of service on the Peninsula defined as in Table 3.

Table 3: Summary Comparison of Run Patterns: Run 35 and Run 48

	Run 35 (Draft Business Plan)		Run 48 (Sensitivity Test)	
Service	Peak	Off Peak	Peak	Off Peak
San Francisco Transbay-SCAG	4	4	2	3
San Francisco 4th and King to SCAG	1	0	1	0
San Jose – SCAG	0	0	2	1
Merced-SCAG	1	1	1	1
Merced-San Francisco	1	1	1	1

Details of the run patterns for both Runs 48 and 35 are in the appendix

The reduction in frequency on the Peninsula is expected to reduce both ridership and revenue by 5 percent (Table 4). For the large LA Basin to Bay Area market, the reduction is expected to be 7 percent, while the market within the Bay Area Peninsula is expected to drop by 12 percent for ridership and 13 percent for revenue.

Table 4: Year 2030 Annual Region-to-Region Ridership and Revenue (millions)
Peninsula Frequency Sensitivity Test
Revenue in 2010 Dollars

	Market	Run 10-035 Phase 1		Run 12-048 Phase 1		Percentage Difference	
		HST Ridership	HST Revenues	HST Ridership	HST Revenues	HST Ridership	HST Revenues
1	LA basin - Sacramento	1.8	$143	1.6	$127	-10%	-11%
2	LA basin - San Diego	0.2	$6	0.2	$6	0%	0%
3	LA basin- Bay Area	8.5	$683	7.8	$633	-7%	-7%
4	Sacramento - Bay Area	0.0	$0	0.0	$0	0%	30%
5	San Diego- Sacramento	0.0	$2	0.0	$2	-33%	-18%
6	San Diego- Bay Area	1.9	$153	1.7	$136	-12%	-11%
7	Bay Area - San Joaquin Valley	5.5	$396	5.2	$376	-5%	-5%
8	San Joaquin Valley - LA basin	5.2	$362	5.2	$360	0%	0%
9	Sacramento - San Joaquin Valley	0.4	$32	0.3	$26	-20%	-18%
10	San Diego - San Joaquin Valley	0.1	$4	0.1	$4	0%	1%
11	within Bay Area Peninsula*	3.3	$59	2.9	$52	-12%	-13%
12	within North LA basin*	3.1	$85	3.1	$85	0%	0%
14	within South LA basin*	1.2	$28	1.2	$28	0%	0%
15	North LA - South LA*	2.8	$78	2.8	$78	0%	0%
18	within San Diego region	0.0	$0	0.0	$0	0%	0%
19	within San Joaquin Valley	0.5	$32	0.5	$32	0%	0%
20	Other	4.6	$293	4.6	$292	-1%	-1%
	Total	39.0	$2,357	37.2	$2,236	-5%	-5%
	within San Diego region	0.0	$0	0.0	$0	0%	0%
	within entire LA basin	7.1	$191	7.1	$191	0%	0%
	within entire MTC	3.3	$59	2.9	$52	-12%	-13%
	within other regions	0.0	$0	0.0	$0	0%	0%
	Total between regions	28.6	$2,107	27.2	$1,993	-5%	-5%

Model Elasticities

Elasticities are econometric measures of the change in demand for a product based on the change in price for that product. For travel forecasting purposes, the change in demand is defined by the change in trips on a specified mode based on the change in one of the factors affecting mode choice[2,3]. For travel demand models, elasticities provide measures of the

2 Appendix B provides a more detailed explanation of elasticities as used for travel demand forecasting.

3 There are several different methods for estimating elasticities. In this memorandum, we have estimated the elasticities using the log-arc formula:

$$\eta = \frac{\Delta LN(Q)}{\Delta LN(P)} = \frac{LN(Q_2) - LN(Q_1)}{LN(P_2) - LN(P_1)}$$

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sensitivity of a model that can be compared with model elasticities from other regions, observed elasticities based on empirical experiments, or simply as reasonableness tests.

Elasticities can be either negative or positive. Negative elasticities mean that demand decreases as "price" increases or vice-versa while positive elasticities mean that the demand changes in the same direction as the price. An elasticity with an absolute value less than 1.0 suggests that the percent change in demand is less than the percent change in price. Models with elasticities greater than -1.0 and less than 1.0 are said to be inelastic; those with elasticities less than or equal to -1.0 or greater than or equal to 1.0 are said to be elastic.

For mode choice models, two types of elasticities can be estimated:

- *Direct-elasticities* measure the change in demand for a mode based on the change in a factor affecting travel on that mode;

- *Cross-elasticities* measure the change in demand for a mode based on the change in a factor affecting travel on a competing mode.

The elasticities reported in this memorandum might be called "aggregate elasticities." They have been based on the change in the total mode share (for high speed rail) versus an overall change in a specific factor affecting mode choice. They have been estimated by comparing the mode shares for an alternative scenario with a changed input assumption to the mode shares for a base scenario without the changed input assumption. Only one input assumption has been changed in each test. For example, high-speed rail travel times for the alternative scenario might have been increased by 10 percent over the travel times in the base scenario for all interchanges. Changes such as removing high speed rail from stations north of San Jose have not been tested since different interchanges would have been affected in different ways. In this example, interchanges served by the San Jose station or stations south of San Jose in the base scenario would have been unaffected by the change while interchanges served by stations north of San Jose in the base scenario would have been greatly affected.

Even though aggregate elasticities have been estimated, elasticities are, by nature, "point" values. The estimated values are dependent on the values of the mode shares and all factors affecting those shares in the base scenario. If "disaggregate" elasticities for two different interchanges were estimated, one with a ninety percent mode share in the base scenario and one with a ten percent mode share, the elasticities would be different even if the change in the input factor for the alternative scenario was the same for the two interchanges. Thus, estimated elasticities for the High Speed Rail model can vary, depending on the scenario used as the base scenario.

Calculated Model Elasticities

Elasticities have been estimated for the High Speed Rail model a number of times. Elasticities were originally estimated for the Peer Review Panel in May 2011 and are documented in a memorandum "Response to Task 3.2 - February 2012 Revision"[4]. Additional elasticities have been estimated based on business plan runs that have changed only one input variable (such as auto operating cost) uniformly for all interchanges or in response to specific elasticity test requests (e.g. high speed rail alternative specific constant ten percent more onerous).

Table 5 summarizes high-speed rail model direct elasticities and cross-elasticities for scenarios that have included the high-speed rail mode. Direct elasticities are shown in ***bold italics***. In some cases, the elasticities have been estimated for business/commute travel and for recreation/other travel separately as well as composite elasticities for all travel. In general, most direct and cross-elasticities from this model are in the inelastic range. There are a few exceptions such as direct elasticities of air travel for recreation/other travel with respect changes in air fares, the direct elasticity of high speed rail travel with respect changes in high speed rail travel time, and the direct elasticity of business/commute high speed rail travel with respect changes in the high speed rail alternative specific constant.

Table 5. Summary of High Speed Rail Model Elasticities

		Mode			
Input Variable Changed	Trip Purpose	Auto	CVR	Air	HSR
Auto Cost Decreased 19 Percent, from 21 Cents/Mile to 17 Cents/Mile (2005 Dollars)	Business / Commute	*-0.04*	0.46	0.20	0.38
	Recreation / Other	*-0.02*	0.44	0.51	0.68
	All Purposes	*-0.03*	0.45	0.43	0.62
Auto Cost Decreased 19 Percent, from 25 Cents/Mile to 17 Cents/Mile (2005 Dollars)	Business / Commute	*-0.03*	0.58	0.20	0.43
	Recreation / Other	*-0.03*	0.55	0.52	0.73
	All Purposes	*-0.04*	0.58	0.35	0.59
Air Fare Decreased by 10 Percent	Business / Commute	0.01	0.00	*-0.39*	0.16
	Recreation / Other	0.01	0.01	*-0.98*	0.44
	All Purposes	0.01	0.00	*-0.68*	0.31
Air Fare Increased by 10 Percent	Business / Commute	0.01	0.00	*-0.44*	0.17
	Recreation / Other	0.01	0.01	*-1.15*	0.45
	All Purposes	0.01	0.00	*-0.77*	0.32
HSR Travel Time Increased by 33 Percent	All Purposes	0.05	0.25	0.70	*-0.97*
HSR Travel Time Increased by 50 Percent	All Purposes	0.05	0.24	0.67	*-1.05*
HSR Cost Decreased by 25 Percent	All Purposes	0.04	0.22	0.37	*-0.53*
HSR Cost Increased by 25 Percent	All Purposes	0.04	0.23	0.44	*-0.71*
HSR Headway Increased by 25 Percent	All Purposes	0.01	0.07	0.12	*-0.22*
HSR Headway Increased by 33 Percent	All Purposes	0.02	0.07	0.12	*-0.22*
HSR Alternative Specific Constant Made 10 Percent More Onerous	Business / Commute	0.06	0.01	0.62	*-2.96*
	Recreation / Other	0.01	0.00	0.04	*-0.16*
	All Purposes	0.03	0.01	0.34	*-1.37*

Source: Cambridge Systematics

4 The memorandum was updated in February 2012 to correct two tables unrelated to the elasticity analysis.

Elasticities from other Sources

Elasticities from a Swiss study were reported in the second report of the Peer Review Panel are shown in Exhibit 1. While the Swiss elasticities are somewhat different than those estimated for the High Speed Rail model, they do have the common characteristic that they are generally in the inelastic range.

Exhibit 1. Reported Elasticities from a Swiss Study

Table 2: Swiss elasticities for long distance travel (Source: Vrtic & Axhausen 2003)

Demand elasticities shown for distances greater than 10 kilometers
(SP parameters at the mean values of the underlying RP trips)

Parameter(s)	Mode	All	Commute	Business	Shopping	Leisure/ Vacation
Travel time car	Car	-0.425	-0.665	-0.68	-0.545	-0.53
	Train/transit	0.671	0.776	1.531	1.008	0.937
Cost car	Car	-0.121	-0.312	-0.076	-0.156	-0.174
	Train/transit	0.191	0.365	0.171	0.288	0.308
In-vehicle-time train/transit	Car	0.365	0.48	0.615	0.46	0.456
	Train/transit	-0.575	-0.56	-1.386	-0.85	-0.805
Fare train/transit	Car	0.157	0.435	0.092	0.223	0.217
	Train/transit	-0.247	-0.508	-0.206	-0.512	-0.373
Access/egress train/transit	Car	0.172	0.272	0.111	0.279	0.127
	Train/transit	-0.272	-0.318	-0.249	-0.515	-0.224
Headway	Car	0.144	0.32	0.154	0.121	0.116
	Train/transit	-0.277	-0.374	-0.346	-0.224	-0.205
Number of travelers	Car	0.115	0.133	0.151	0.101	0.134
	Train/transit	-0.181	-0.156	-0.339	-0.186	-0.237

Source: Independent Peer Review of the California High-Speed Rail Ridership and Revenue Forecasting Process, Findings and Recommendations from April-July 2011 Review Period, August 1, 2011.

Extreme Downside Scenario

This scenario was intended to test the ridership and revenue implications of a whole series of downside events lining up at the same time. We tested these events using the Initial Operating Segment phase since the Business Plan showed that the finances were most fragile at this time.

This scenario was compared to the IOS scenario used in the Revised Business Plan. Table 6 summarizes the difference in assumptions between the Revised Business Plan (Run 41d) and the Extreme Downside Scenario (Run 53.) Since the Revised Business Plans expressed dollars at the 2011 level, this comparison does so as well.

Table 6: Extreme Downside Case Assumptions
Compared to IOS in Revised Business Plan

	Revised Business Plan Run (12-041d)	Extreme Downside Scenario (Run 12-053)
North Terminal	Merced	Same
South Terminal	San Fernando	Same
HST Fare Policy	83% of San Francisco-Los Angeles airfares with lower rates for shorter distances	Same
CVR Fare Policy	Actual 2011	Same
Socioeconomic Forecast	Based on 2011 Moody's Analytics Forecast for 2030	Same
Trip Rate	2011 Survey	Same
Conventional Rail Connections	Existing Amtrak San Joaquin service terminates at Merced (service to Bakersfield discontinued	Same
Auto Operating Cost (2011$)	20 cents / mile	18 cents / mile
Air Fare Policy	Actual 2009	10% Less than actual 2009
HSR Summary	4 peak TPH (2 in offpeak)	3 peak TPH (2 in offpeak)
Dedicated Peak Coach Service	South: • 4 peak BPH from San Fernando to LAUS, to West LA, and to Santa Anita North: • 4 peak BPH from Merced to Sacramento, to San Francisco, and to San Jose	South: • 3 peak BPH from San Fernando to LAUS, to West LA, and to Santa Anita North: • 3 peak BPH from Merced to Sacramento, to San Francisco, and to San Jose
HST Average Travel Time (Merced-San Fernando)	126 minutes	140 minutes
HST constant in mode choice model	Original	10 percent lower than original

The Extreme Downside case would be expected to reduce ridership for the IOS to 4.7 million riders per year, 27 percent below that shown for the low scenario in the Revised Business Plan (Table 7). The revenue for this scenario is expected to be $338 million per year, which is 28 percent below the IOS low case in the Revised Business Plan.

Table 7: Year 2030 Annual Region-to-Region Ridership and Revenue (millions)
Extreme Downside Scenario of Initial Operating Segment (IOS)
Revenue in 2011 Dollars

		Run 12-041d IOS		Run 12-053 IOS		Percentage Difference	
	Market	HST Ridership	HST Revenue	HST Ridership	HST Revenue	HST Ridership	HST Revenue
1	LA basin - Sacramento	0.2	$19	0.1	$12	-42%	-40%
2	LA basin - San Diego	0.1	$1	0.0	$1	-20%	-18%
3	LA basin- Bay Area	1.2	$97	0.7	$55	-44%	-44%
4	Sacramento - Bay Area	0.0	$0	0.0	$0	0%	0%
5	San Diego- Sacramento	0.0	$0	0.0	$0	0%	-57%
6	San Diego- Bay Area	0.0	$2	0.0	$1	-67%	-55%
7	Bay Area - San Joaquin Valley	0.2	$10	0.1	$7	-33%	-32%
8	San Joaquin Valley - LA basin	3.8	$267	3.1	$214	-19%	-20%
9	Sacramento - San Joaquin Valley	0.0	$2	0.0	$1	-33%	-46%
10	San Diego - San Joaquin Valley	0.0	$0	0.0	$0	-100%	-28%
11	within Bay Area Peninsula*	0.0	$0	0.0	$0	0%	0%
12	within North LA basin*	0.7	$20	0.7	$20	-2%	-1%
14	within South LA basin*	0.0	$0	0.0	$0	0%	0%
15	North LA - South LA*	0.0	$0	0.0	$0	0%	0%
18	within San Diego region	0.0	$0	0.0	$0	0%	0%
19	within San Joaquin Valley	0.1	$6	0.1	$4	-38%	-36%
20	Other	0.7	$59	0.5	$43	-28%	-28%
	Total	7.1	$486	5.3	$357	-25%	-26%
	within San Diego region	0.0	$0	0.0	$0	0%	0%
	within entire LA basin	0.7	$20	0.7	$20	-2%	-1%
	within entire MTC	0.0	$0	0.0	$0	0%	0%
	within other regions	0.0	$0	0.0	$0	0%	0%
	Total between regions	6.4	$466	4.7	$338	-27%	-28%

Probability of the Extreme Downside Case

The analysis above demonstrates that the Extreme Downside case would result in revenues about 28 percent lower than the IOS Low scenario. While enlightening, this does not tell us how likely the Extreme Downside case is to occur.

To estimate the probabilities of the downside would require estimating the probabilities of the factors that make up that scenario occurring. Table 8 shows the probabilities that would need to be estimated. Ideally, these probabilities would be estimated by a group of independent experts representing different topical specialties, with the aim of achieving consensus, perhaps through a Delphi technique. This analysis is being carried out at the present time and the results will be presented in an addendum to this memorandum. As an illustrative example, however, we have populated the table with extremely conservative assumptions regarding the probability of each of these events occurring. The product of these probabilities is 5 percent.

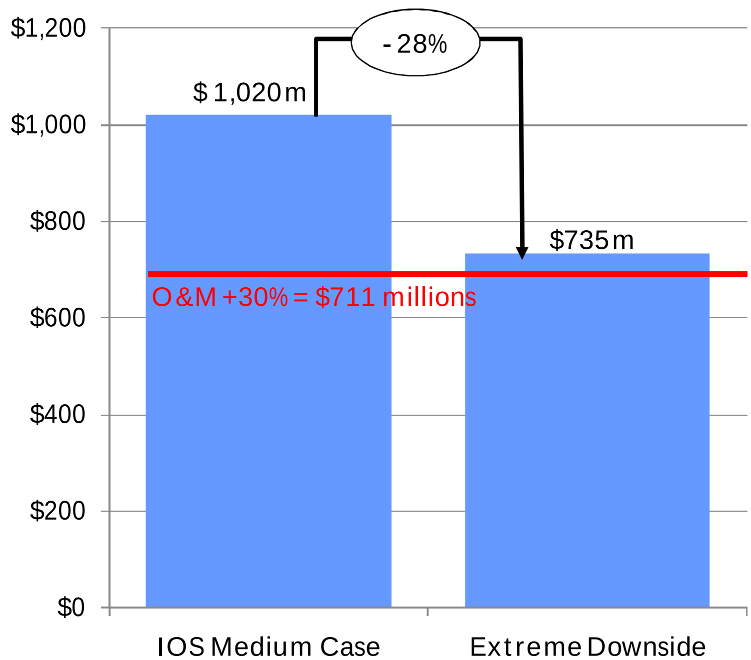
Table 8: Probability of Individual Components of Extreme Downside Occurring
Template for Analysis and Illustrative Probability

Risk factor	Illustrative Probability
Airfare 10% lower than 2009 levels (real terms), or lower	45%
Motor fuel costs at 18 cents per mile in 2011 dollars, or lower. The current high and low range for the BP forecasts is 20 cents, which was based on $2.60/gallon gas and 27 mpg. The 18 cents put together the low gas price and high fuel efficiency assumption ($2.60 and 33.6 mpg)	45%
High speed rail constant reduced by 10%, or lower	45%
10 percent increase in train run times	75%
Reduce peak trains per hour from 4 to 3	75%

Impact on Profitability and Breakeven

The Extreme Downside Case shows an impact of -28% on total revenue. Applied to the IOS medium case in 2026, this would result in a drop from $1,020 million to $735 in YOE dollars, as shown in Exhibit 2. In the same year, total Operating & Maintenance costs are forecasted to be $547 million in YOE. Even with an O&M expense sensitivity of 30%, the system still shows a positive cash flow from operations of about $25 million in 2026.

Exhibit 2. Extreme Downside Case and 30% Increase in O&M Costs in 2026 – in millions of $YOE



The breakeven analysis is detailed in Chapter 7 of the Revised Business Plan, Funding & Financing. The analysis identifies the revenue necessary to balance the minimum operating and maintenance costs needed to run the system.

A final sensitivity test was performed on the revenue and O&M projections to compare the results with the breakeven forecast. In this analysis a -35% impact was applied to the IOS medium case. It is important to note that the Extreme Downside Case forecasted a decrease of 28% on revenue only. To simulate an increase of 35% in O&M costs, we applied an equivalent decrease in revenues. This allowed us to have a direct comparison with the breakeven projections determined in the Revised Business Plan. Table 9 presents a comparison of the results with the breakeven projections for 2022 through 2030.

Table 9: Revenue and Breakeven projection (-35% revenue and +35% O&M) in millions of $YOE

	2022	2023	2024	2025	2026	2027	2028	2029	2030
Revenue	$207	$286	$370	$460	$557	$814	$926	$1,140	$1,305
Breakeven	$218	$225	$231	$238	$245	$253	$260	$268	$276
Profit / Loss	($11)	$61	$139	$222	$311	$561	$666	$872	$1,029

Source: Parsons Brinckerhoff Analysis based on IOS Medium Case Revenue Projections

In 2026, once the system has reached steady state operation (post ramp-up) and under this extreme range of sensitivities, the system generates over $300 million positive cash flow over breakeven.

As mentioned earlier, the calculation of the probability of the Extreme Downside case is currently in progress. The illustrative example shown in Table 8, while using very conservative examples, gives a sense of the improbability of the Extreme Downside Case. A combined increase of 35% of the O&M expenses with the Extreme Downside Case further reduces the possibility of such an event.

Appendix A

Model Run Output Summaries

- Run 35: Draft Business Plan Phase 1
- Run 49: Sensitivity Test, Phase 1, 50 MPG
- Run 48: Sensitivity Test, Phase 1, Reduced frequency on Peninsula

- Run 41b: Revised Business Plan, IOS
- Run 53: Extreme Downside Case, IOS

Appendix B

Elasticity Discussion and Formulation

(Source: TCRP Report 95, Appendix A)

Appendix A — Elasticity Discussion and Formulae

THE ELASTICITY CONCEPT

Elasticity is a convenient quantitative measure of travel demand response to price and service changes which influence demand. Elasticity measures are found throughout the transportation literature and have been reported and used in various sections of this Handbook. When used with caution, elasticities provide a satisfactory means of quickly preparing first-cut, aggregate response estimates for a number of types of system changes. When considering demand for transportation, there are a number of elasticities of interest, including elasticities describing traveler response to changes in the overall amount of transit service, transit frequencies, transit fares, vehicular tolls, parking charges, and gasoline costs.

For elasticity measures to be applicable, the transportation system change must be a relative one. In other words, it must involve a quantifiable percentage increase or decrease in the system parameter involved. For example, while elasticity measures can be used to describe the response to a change in the overall amount of transit service, they cannot be used to describe the response to a new bus system.

Transportation elasticities are informally adopted from the economist's measure "price elasticity." The price elasticity of demand is loosely defined as the percentage change in quantity of commodity or service demand in response to a 1 percent change in price. For instance, a price elasticity of -0.3 indicates that for a 1 percent increase (decrease) in the price of a good or service, there is a 0.3 percent decrease (increase) in the demand for that good or service.

It would be more precise to say, however, that a price elasticity of -0.3 indicates an 0.3 percent reduction (increase) in demand in response to each 1 percent price increase (decrease), calculated in infinitesimally small increments. (The order of the statement is not important, but the calculation in infinitesimally small increments is.)

The negative sign signifies an inverse relationship between price and demand. In other words, it indicates that the effect operates in the opposite direction from the cause. For example, an increase in price results in a decrease in demand, and the corresponding elasticity is negative. An increase in service promotes an increase in demand, and the elasticity is positive.

If a 1 percent change in a parameter causes a greater than 1 percent change in demand, demand is said to be elastic. If a 1 percent change in a parameter causes a less than 1 percent change in demand, then demand is said to be inelastic. Many, but not all, transportation system changes elicit responses that are so-called inelastic.

MEASURES OF ELASTICITY

There are three different methods commonly found in the transportation literature for computing elasticities:

- Point elasticity
- Arc elasticity
- Shrinkage factor

Point elasticity is derived directly from the economist's definition of elasticity. Mathematically, it is described by the following formula:

$$\eta_P = \frac{dQ}{dP} x \frac{P}{Q}$$

where η_P is the elasticity at price P, and Q is the quantity demanded at that price.

In practice, lack of information on the functional relationship between P and Q (the shape of the demand curve) precludes the computation of point elasticities from empirical data. Therefore, other formulations have been developed which allow the use of observed changes in price and associated demand.

The measure which most nearly approximates point elasticity, and one frequently employed, is arc elasticity. It is defined by a logarithmic formulation and, except for very large changes in P and Q, is closely approximated by a mid-point (or linear) formulation which makes use of the average value of each independent variable (Bly,1976; Mayworm, Lago and McEnroe, 1980).

log arc elasticity:

$$\eta = \frac{\Delta \log Q}{\Delta \log P} = \frac{\log Q_2 - \log Q_1}{\log P_2 - \log P_1}$$

mid-point (or linear) arc elasticity:

$$\eta = \frac{\Delta Q}{(Q_1 + Q_2)/2} \div \frac{\Delta P}{(P_1 + P_2)/2} = \frac{\Delta Q(P_1 + P_2)}{\Delta P(Q_1 + Q_2)} = \frac{(Q_2 - Q_1)(P_1 + P_2)}{(P_2 - P_1)(Q_1 + Q_2)}$$

where η is the elasticity, Q_1 and Q_2 are the demand before and after, and P_1 and P_2 are the price or service before and after.

Arc elasticity is based on both the original and final values of demand and price or service. When one of the values is zero, as in the case of adopting or terminating free transit, the midpoint arc elasticity formulation must be employed. Otherwise, the logarithmic formulation has been used whenever elasticities have been calculated directly from available data in this Third Edition Handbook, as was the case with the Second Edition. Similar values carried over from the First Edition were computed using the mid-point formulation.

A third form of elasticity, historically used in reporting response to transit fare changes, is the shrinkage factor or shrinkage ratio. In its general use "rule of thumb" formulation, it is defined as the change in demand relative to the original demand divided by the change in price relative to the original price, or in mathematical terms:

$$\eta = \frac{\Delta Q / Q_1}{\Delta P / P_1} = \frac{(Q_2 - Q_1)/Q_1}{(P_2 - P_1)/P_1}$$

Shrinkage factors present certain conceptual difficulties. For example, consider a specific experimental transportation price reduction or service expansion and the accompanying travel volume increase. Assume, for illustrative purposes, that the demand returns to its original level if the price is raised or the service reduced back to its original state as a second experiment. Logically, the elasticity in this hypothetical example should be the same for both experiments, and it is — if arc elasticity is computed. However, if the changes in price or service are moderately large, the corresponding shrinkage factors will be different. Shrinkage factor guidelines that are in common use are reported in this Handbook, but arc elasticity conversions are given where possible.

Note that this generalized "rule of thumb" formulation for shrinkage factors is not the version derived and applied by the firm of Simpson and Curtin for describing and predicting transit fare change impacts. That formulation included a constant (Curtin, 1968), as described and examined in full in Chapter 12, "Transit Pricing and Fares," under "Response by Type of Strategy" — "Changes in General Fare Level" — "Urban Transit Overall." The Simpson and Curtin formulation has the same conceptual problems as described above, however.

DIFFERENCES BETWEEN ELASTICITY MEASURES

When the percentage change in price or service is small, all the methods for computing elasticity give approximately the same value. Large changes, however, result in different values of elasticity depending on the formula used. Table A-1 gives elasticity values calculated for different fare changes and an assumed log arc elasticity of -0.300.

Table A-1 Values of Elasticity According to Different Methods of Computation

Percent Fare Change	Log Arc Elasticity	Mid-Point Arc Elasticity	Shrinkage Factor
-50%	-0.300	-0.311	-0.46
-30	-0.300	-0.303	-0.38
-10	-0.300	-0.300	-0.32
+10	0.300	0.300	0.28
+30	-0.300	-0.302	-0.25
+50	-0.300	-0.304	-0.23
-100	-0.300	-0.311	-0.19

Figure A 1 illustrates the differences in the three measures of elasticity for an initial point price elasticity of -0.30 (Mayworm, Lago and McEnroe, 1980).

For both point and arc elasticities, an absolute elasticity value greater than 1.0 signifies an elastic relationship, while an absolute value less than 1.0 indicates an inelastic relationship. This is not necessarily the case for the shrinkage ratio, as the transit fare reduction example below illustrates (Dygert Holec and Hill, 1977). The loss of revenue in the example shows that increased ridership was not great enough to offset the fare decrease in terms of revenue. This illustrates an inelastic relationship between fare and ridership.

Initial fare = $0.40	Initial ridership = 1,000	Initial revenue = $400
Final fare = $0.25	Final ridership = 1,500	Final revenue = $375

Shrinkage ratio = -1.33
Log arc elasticity = -0.86

USE OF ELASTICITIES IN THE HANDBOOK

Elasticities should not be taken or used as precise predictive measures. They simply serve to indicate the likely order of magnitude of response to system change, as inferred from aggregate data on the experience in other, hopefully comparable, instances. However, they can be very useful in providing first-order estimates of the changes in demand which may be expected for certain price or service changes.

Elasticity Application Formulae

The formulae for applying arc elasticities to predict traveler response are not the same as for applying shrinkage factors. Given a proposed transportation system change, to compute the new travel demand which may be expected given an arc elasticity value thought to be applicable, the equations to use are:

log arc elasticity:

$$Q2 = 10^{\eta\,(\log P_2 - \log P_1) + \log Q_1}$$

mid-point (or linear) arc elasticity:

$$Q_2 = \frac{(\eta - 1)P_1Q_1 - (\eta + 1)P_2Q_1}{(\eta - 1)P_2 - (\eta + 1)P_1}$$

where η is the arc elasticity, Q_1 and Q_2 are the demand before and after, and P_1 and P_2 are the price or service before and after.

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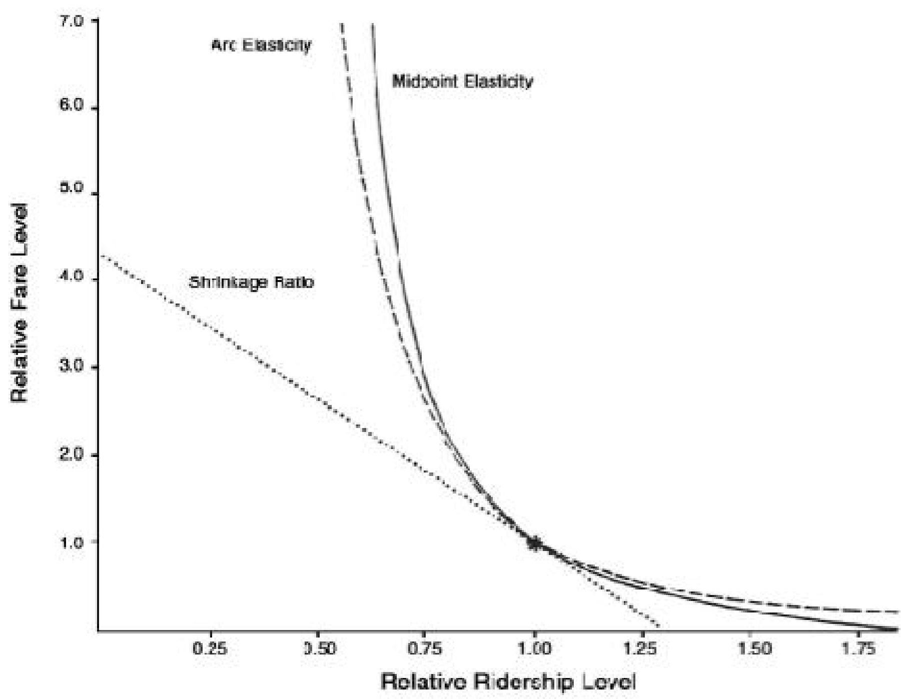
Figure A-1 Elasticities of Different Types Calculated from a Demand Curve with an Initial Point Elasticity of –0.30



Note: The term "point elasticity" as used in the figure title refers to the derivative of the assumed underlying demand curve — it is not used here as a synonym for shrinkage ratio or factor.

Source: Mayworm, Lago and McEnroe (1980).

A-5

Following is an example of arc elasticity application:

Assume that a transit operator with a daily ridership of 21,000 (Q_1) is interested in increasing fares from 35¢ (P_1) to 45¢ (P_2), and that the applicable fare elasticity (η), arc formula, is -0.40. The new ridership (Q_2), which could be expected following the fare increase, as estimated using fare elasticity, would then be computed as shown:

log arc elasticity:

$$Q_2 = 10^{-0.4\,(log\,45\ -\ log\,35)\ +\ log\,21{,}000} = 19{,}000$$

mid-point (or linear) arc elasticity:

$$Q_2 = \frac{(-0.4 - 1.0)(35)(21{,}000) - (-0.4 + 1.0)(45)(21{,}000)}{(-0.4 - 1.0)(45) - (-0.4 + 1.0)(35)} = 19{,}000$$

Thus, the estimated decrease in daily ridership would be 2,000 passengers.

Source material constraints have precluded exclusive use of arc elasticities (or the closely comparable point elasticities) in this Handbook. Where elasticities derived using other formulations are given, the type is indicated, if known.

Elasticity Definitional Differences

The reader must be alert to major elasticity definitional differences among this Handbook and other references, older ones in particular. Table A-2 illustrates various extant definitional differences with respect to elasticity:

Table A-2 **Definitional Differences with Respect to Elasticity**

Handbook	Mayworm, Lago and McEnroe (1980)	Bly (1976)	Dygert, Holec and Hill (1977)
shrinkage ratio	shrinkage factor	shrinkage ratio	arc elasticity*
fare or service elasticity* (or log or mid-point arc elasticity)	arc elasticity* (log or mid-point)	fares elasticity* (or arc, or linear arc elasticity)	"Kemp…definition of arc elasticity"
point elasticity*	point elasticity*	point elasticity*	point elasticity*

Note: The forms principally used in the respective publications are indicated by an asterisk (*). Note that the discussions of arc elasticity properties in Dygert, et al pertain only to what are termed shrinkage ratios/factors or growth ratios/factors elsewhere.

In addition to the definitional differences listed in Table A-2, the user of elasticities also needs to be aware that shrinkage ratios or factors are sometimes called point elasticities. This confusion pervades even textbooks. A true point elasticity uses the derivative of the demand curve, which is the slope for the entire demand curve or function. One must have a mathematical function to work from in order to derive a true point elasticity, which is not the case with raw quasi-experimental data. When point elasticity nomenclature is applied to what is otherwise referred to as a shrinkage ratio or factor, it is simply the elasticity for the demand curve at one particular point on the curve, irrespective of whether the whole curve is known or not. The problem with this method is that the elasticity is different at different points on the curve, causing the conceptual deficiencies noted earlier for shrinkage ratios. This limitation has led to growing acceptance of log or mid-point arc elasticities as the preferred approach for use with quasi-experimental data. Arc elasticities apply not to a single point, but to the entire portion of the demand curve under study.

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